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Aerodynamics of Spheres

Lawrence Lemke
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Introduction

This discussion is a qualitative treatment of a topic that is usually treated rigorously and quantitatively in engineering textbooks. This discussion is intended for the general audience of nonspecialists and will attempt to convey the main ideas behind aerodynamics of spheres in a qualitative manner, with a minimum number of mathematical equations. As the Latin roots of the word imply, aerodynamics deals with the dynamic flow of air (or any gas that acts like air) when in proximity to a solid body. Aerodynamics is a special subset of the more general topic of fluid dynamics. The significant distinction between air and more generic fluids is that air is a compressible fluid, whereas other fluids such as water may not be. As a compressible fluid, the density of air may vary from point to point in space as it interacts with a solid body and flows around it.

Discussion

A fluid is sometimes defined as a homogeneous body of matter that coheres together but has no intrinsic shape of its own; rather, it takes the shape of its container. Since the time of the early 19th century it has been understood that a body of fluid is an extremely large collection of independent molecules in continuous kinetic motion. The molecules are very loosely bound to each other and are continuously colliding with each other and with the surfaces of solid bodies in contact with the fluid. As the molecules collide they transfer kinetic energy with each other and with the solid surfaces. This transfer of kinetic energy gives rise to all of the forces that are experienced by the body and by the fluid itself.

This mutual transfer of kinetic energy means that aerodynamics may be viewed from two viewpoints. From the viewpoint of the fluid itself, the challenge is to explain the motion of all particles making up the fluid. From the viewpoint of the solid body, the challenge is to explain the forces generated by the fluid flow and imposed on the body. For a complete scientific understanding of aerodynamics both viewpoints must be considered. From a practical standpoint, it is often the net effect of airflow on a body that is important. For example, the amount of lift generated by a wing is the main concern that determines whether it will act as a lifting body and therefore whether an airplane will fly or not. In this discussion, we will be mainly concerned with the net effect of aerodynamic forces on spherical bodies, and we will discuss only as much of the details of the fluid motion as is necessary to explain these effects.

Even though the patterns in fluid flow and the forces exerted on solid bodies in contact with the fluid arise from the continual collision of astronomical numbers[1] of discrete molecules, it is fortunate that it is possible to understand fluid flow in a systematic manner without the necessity of attempting to keep track of such numbers of interactions—which is a practical impossibility in any case. This is accomplished by introducing abstract variables such as pressure, density, and the speed of sound which exist as ensemble average values, measurable at any point in the fluid. Instead of attempting to describe the motion of a practically infinite number of individual particles, aerodynamics explains the behavior of a small number of flow variables.

As a practical matter, aerodynamics may be divided into two different types of flow conditions. One condition is internal air flow in enclosed ducts, pipes, and the like; the other condition is external air flow around a free body. In this discussion we are concerned with the latter case—a single sphere moving through a uniform body of air. It is customary in aerodynamic analysis to consider the solid body as being stationary in the reference frame with a large mass of air flowing past it at some reference velocity (v). This velocity is referred to as the free stream velocity. The assumption of uniformity means that the body of air is assumed to be so large relative to the sphere that there is no interaction between air flow around the sphere and air flow from boundaries or other objects. Also, there is no variation in the free stream velocity in the direction lateral to the flow velocity before the flow contacts the sphere.

Like all vector quantities, the free stream velocity is characterized by both a magnitude and a direction. In general, when an air flow begins at a great distance from a body and encounters the body[2], the forward facing surfaces of the body will force the flow to diverge in a direction laterally outward relative to the free stream velocity. In common terminology, the body pushes the air out of the way to the side. As the flow is forced laterally, its velocity increases. After the flow passes the point corresponding to the maximum cross-section area of the body the flow will tend to converge laterally inward back toward the original flow direction. This makes the important point that the air velocity immediately adjacent to the skin of a solid body may be speeding up or slowing down relative to the free stream velocity. These differences in velocity from point to point on the body correspond to differences in air density and pressure at those points.

If one were able to make a time-lapsed photograph of an individual molecule in the flow field as it first approached, encountered, and then passed by the body, one would see it describe a continuous path. This continuous path--no matter how twisted and complicated it may be--is referred to as a streamline. It represents the natural path a molecule (or small body of fluid) follows as it moves from the direction of high pressure in the direction of low pressure. The pressure field varies from point to point in the air mass and therefore molecules located at slightly different positions in the air mass will follow slightly different paths. The sum total of

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1 Avogadro's number, or approximately 6×10^{23} is the number of molecules in 29 grams of air.

2 Whenever the term "body" is used a rigid and solid body is assumed.

all of these paths represents a conceptualization of the flow field around the body.

It is an empirical fact that there will be one point on the body where the streamlines separate. This may be thought of as analogous to the highest point on a mountain that separates two different watersheds. A raindrop falling on one side of this peak will end up in one river; a raindrop falling on the other side of the peak will end up in an entirely different body of water. This point on the body that separates streamlines is referred to as the stagnation point. It has the characteristic that an air molecule coming in with the free stream velocity and hitting the stagnation point will come completely to rest (zero velocity). Points on the surface nearby the stagnation point will experience air flow whose velocity is reduced relative to the free stream velocity, but not completely to zero. Air flow that is very far from the stagnation point and does not impinge the body at all will have its velocity reduced by a very small (even negligible) amount. When an air mass has its velocity reduced from some initial value, the kinetic energy of the air is turned into pressure. Because this pressure derives from the motion of the air, it is customarily referred to as dynamic pressure and represented by the variable, Q. In the case where the freestream velocity (***V***) of the molecule is reduced to zero (at the stagnation point), the dynamic pressure is numerically equal to one-half the density of the air (*p*) times the velocity (v) squared (Eq. 1)

$$Q = \frac{1}{2} \rho V^2 \qquad (1)$$

It can be seen, from this simple discussion that Q will be maximum at the streamline impacting the stagnation point, will transition to smaller values on streamlines on the body away from the stagnation point, eventually falling to zero for streamlines not contacting the body.

The dynamic pressure acting on a small increment of surface area of the body exerts a force. Customarily, the component of this force that acts in the same direction as the free stream velocity of the air is referred to as *drag*. The component of this force that acts at right angles to the velocity direction (if any) is referred to as *lift*. In principle, if one adds up all the incremental forces generated by the dynamic pressure at every point on the surface of the body one will get the total resultant force acting on the body due to dynamic pressure. This illustrates the important point that aerodynamic forces are surface forces. That is, they are transferred from the mass of moving air to the body through a surface. Therefore, in order for the conclusions of this discussion to be valid it is important that the body in question be characterized by a solid, impermeable exterior skin. In other words, a required condition is that no fluid flows through the exterior skin, but only around it.

A given rigid body subject to aerodynamic analysis can be characterized by its shape, its size, and its orientation relative to the free stream velocity. The orientation is often referred to as the angle of attack. It is intuitively obvious that a single object for example, a flat plate, will produce a very different flow field if it is presented to the air flow with its thin edge-on instead of broad side on. Any child who has held his hand out an automobile window while it was moving at speed and rotated it back and forth it will have direct experience of this fact. It is a fortunate empirical fact that, for a given body shape, size, angle of attack, and air velocity the flow field around the body will be geometrically similar more or less independent of the size of the body. This statement is not exactly true, but is approximately true over a wide range of body sizes and air velocity and is therefore a useful concept. It produces the fortuitous result that if the flow field and forces around an object of one size can be determined then the flow field and forces around a similar shaped object of a different size moving at a different speed may be inferred with some confidence. This generalization is what makes aerodynamics a tractable field of engineering. This allows, for example, the performance of a large airplane to be predicted on the basis of testing a small model of that airplane in a wind tunnel.

We will now apply this principle to airflow around spheres. In aerodynamics, a sphere is practically the simplest shape to analyze. As discussed above, the flow field around an object is strongly affected by its angle of attack. A sphere has symmetry around all three axes, and therefore its angle of attack never varies. In effect, the angle of attack of a sphere is always zero. Nevertheless, even with exceptionally simple geometry the flow around a sphere can be complex for several reasons.

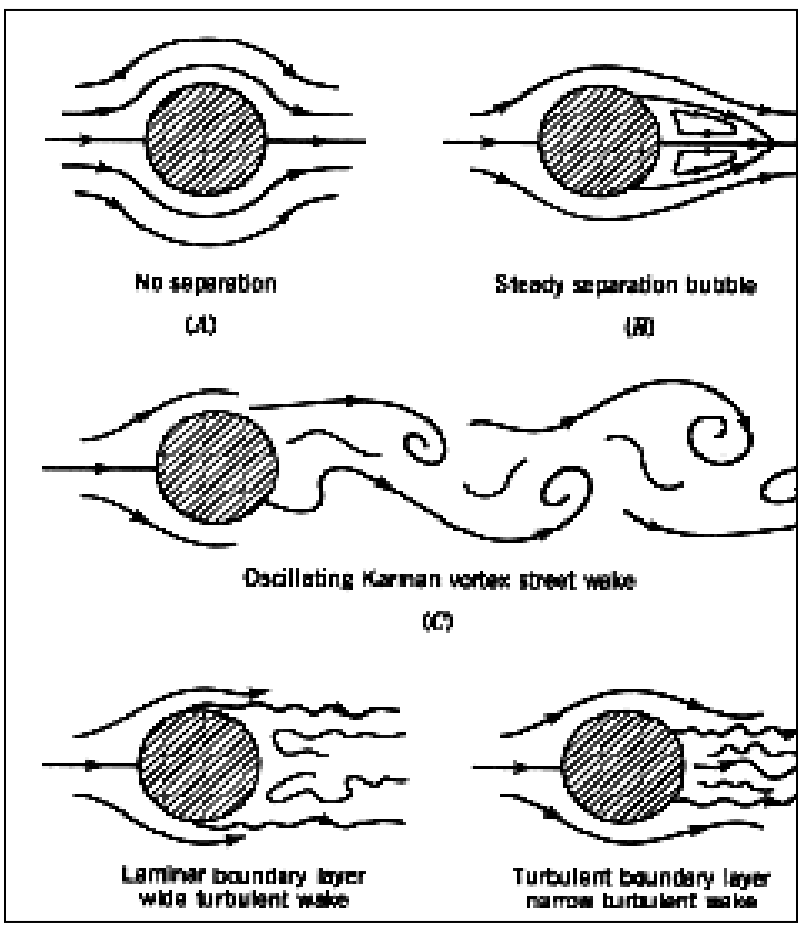
At the microscopic scale where individual air molecules interact with the skin of the body, there are competing physical processes affecting the motion of the air molecules. Because the air molecules encounter the body with an initial velocity, they have momentum. That is, air molecules like all massive bodies in motion have a tendency to want to remain in motion according to Newton's laws. At the same time, air molecules are attracted to the molecules of the skin through electronic forces. These electronic forces appear in the form of viscosity. Viscosity is a force acting between two small elements of mass moving relative to each other; if one of the elements in question is the surface of a rigid body then the direction the viscous force is acting in may be different than the direction the momentum force is acting in. At low speeds, the exact direction that air masses move in at any point in the flow field around an object is determined mainly by the balance between momentum and viscosity forces. This balance changes with gas composition, density, size of the object, and velocity. The importance of this balance between competing factors was first recognized by the scientist Gabriel Stokes in the 1850s and popularized by the scientist Osborne Reynolds in the 1880s. A quantitative measure of this balance between viscosity and momentum is referred to as the Reynolds Number. Numerically, the Reynolds Number is defined as:

$$\mathrm{Re} \equiv \frac{\rho v x}{\mu} \qquad (2)$$

where p is the fluid density, v is the flow velocity, x is a characteristic dimension of the object (the sphere's radius, for example) and μ is the dynamic viscosity.

We will now consider qualitatively how the Reynolds Number affects aerodynamic flow around a sphere. First, consider flow with a low Reynolds Number; a very small sphere moving slowly--such as a falling raindrop--is a good example. At low speeds, the momentum of an air molecule is low. This means that the attractive force between an air molecule and a water molecule on the surface of the drop is relatively large in comparison. Therefore, the momentum that would normally cause the air molecule to try to travel in a straight line is overcome by the attractive force and as a result the streamlines are attached very closely and conform to the spherical shape. In other words, the shape of a streamline immediately adjacent to the surface is a circular arc. Whenever a streamline shape conforms very closely to the surface shape immediately adjacent to it, the flow is essentially parallel to the surface and we refer to this as attached flow or laminar flow. For purely laminar flow around a sphere, the streamlines will separate radially from around the stagnation point on the nose and rejoin each other at a point located 180° away on the sphere's rear surface [Fig. 1 (A)]. If the fluid surrounding the sphere had no viscosity, it can be shown mathematically that the pressure distribution on the back half of the sphere would be equal and opposite to the pressure on the front half and that therefore the sphere would have no resistance to motion or no drag. In fact all real fluids including air have viscosity and for small spheres moving slowly essentially all of the drag originates from a viscous fluid flowing parallel to the sphere's surface.

At the molecular level, viscosity acts to keep the streamlines attached to the surface; as the velocity (and therefore momentum) increases air molecules have a stronger tendency to remain moving in the direction of the free stream velocity. As the streamlines cross over the equator between the front half and back half of the sphere they detach from the surface and no longer converge at a single point opposite the stagnation point. This is called detached flow. Reynolds was the first scientist to recognize and systematically explore the phenomenon that, when the flow reached a critical balance between momentum and viscous forces (what is today referred to as the critical Reynolds Number) the flow will transition from laminar to turbulent. When this transition occurs on a sphere, there will first be a large region on the back half of the sphere in which a stable circulating vortex will form [Fig. 1(B)]. As the Reynolds Number increases further, the point at which flow detaches moves from forward on the back half of the sphere until it reaches the equator that separates the front and back halves of the sphere. Under this condition, a stable circulating flow it is not possible, and the detached flow will enter an oscillating or unsteady regime. When observed in a time lapse manner, it will appear that vortices build up in magnitude and then separate and are carried downstream in a repeating pattern. In this condition, the sphere is said to be "shedding vortices" [Fig. 1 (C)].



At still greater Reynolds Numbers the process of vortex shedding will become steady

Figure 1. (Previous Page)Flow field around a sphere becomes more complex and turbulent as Reynolds Number increases. (Figures reproduced from Princeton University Department of Engineering lecture notes)

and the flow will be characterized by laminar flow on the front half of the sphere and turbulent behind the back half of the sphere. At the highest subsonic Reynolds Numbers, the flow will finally become turbulent on both the front half of the sphere and in the detached wake behind the sphere.

As the free stream velocity (v) approaches the speed of sound in air (c), another complicating factor arises; this factor is the compressibility of air. The ratio of these two velocities is given the name, Mach Number, defined as:

$$M = {}^{v}\!/_{c} \qquad (3)$$

Like the Reynolds Number, the Mach Number is a dimensionless number which—when it increases to certain values—signifies the onset of different flow regimes. The physical significance of the speed of sound is that it is the average velocity with which the fluid molecules are moving in random directions. Thus, as the free stream velocity approaches M=1, it approaches the average random velocity of fluid molecules. When the local airflow around a solid surface reaches or exceeds this average velocity, air molecules cannot move fast enough to get out of the way of the surface and they pile up. This process of piling up creates local regions of very high pressure and high density. These regions are referred to as shockwaves. As discussed above, when air flow encounters a solid body such as a sphere, flow separates from the stagnation point and speeds up as it is forced to flow around the front half of the body. Thus, the local speed of air flow at some point on front half of the body will become significantly greater than the free stream velocity. Therefore, as the free stream velocity is increased toward the speed of sound, air speed around the spherical surface will reach and exceed M=1 somewhere before the free stream velocity does. When the airflow at some point on the body becomes close to or slightly exceeds M=1, the airflow is referred to as transonic. Under transonic conditions, the flow regimes described above, which were driven primarily by momentum and viscosity are now modified by the presence of compressibility driven shock waves.

When the air velocity everywhere on the sphere exceeds the speed of sound the flow is said to be supersonic. Truly supersonic flow will be characterized by the presence of stable shockwaves located in a fixed relationship to the nose and the trailing surface of the sphere. Air that flows through the shockwaves will experience extreme compression and therefore heating. At low supersonic conditions, the compressed and heated air is capable of transferring significant amounts of heat to the object it impinges upon but the nature of the air itself is not transformed. As an object such as a sphere moves through the transonic range and up through the supersonic range the geometric arrangement of shockwaves around the sphere can be observed to change. In general, as the flow becomes faster the shockwave angle will become more swept back and it's attachment point on the sphere will move rearward. At speeds above approximately M = 5 the pattern of shockwaves will be seen to converge to a stable configuration. In other words, the flow pattern once again becomes simple. Velocities above this approximate speed are referred to as hypersonic, and this tendency to converge to simpler flow conditions is referred to as “hypersonic invariance”. Thus, in a very general sense airflow at very low subsonic conditions and airflow at hypersonic conditions is relatively easy to describe. It is in the middle or transonic range that flow conditions are most complex. At very high hypersonic speeds, compression heating can be so severe that the individual air molecules are broken apart and ionized; in this case the flow will react to electromagnetic fields and become extremely complex, once again.

As can be seen, a full description of the details of airflow around a sphere can be very complicated and will depend critically on the size and velocity of the air flow. Nevertheless, the net effect on the sphere is rather simple; *it results in drag but no lift*. In other words, because of the symmetry of the sphere, all the complicated forces and effects of momentum, viscosity, and compressibility simply create a net force tending to drag the sphere in the direction of the free stream velocity. The magnitude of the drag force will change along with the details of velocity, size, and even surface smoothness, but its direction will always be parallel to the free stream velocity. One way of conceptualizing this for purposes of quantifying the drag force is by introducing the idea of a non-dimensional number referred to as a drag coefficient. As we discussed above, when air with a free stream velocity, v, is brought completely to rest it produces a dynamic pressure, (Q) (Eq.1). Any solid body has a certain cross-sectional area, (A), that it presents perpendicular to the velocity. For a sphere the cross-sectional area is simply the area of a circle with the same radius as the sphere ($A = \Pi r^2$). The cross-sectional area (A) represents the maximum possible area of the body that air could intercept. The dynamic pressure (Q) represents the maximum possible pressure that any piece of area could experience. Since Q has units of pressure and A has units of area, multiplying Q times A yields a product that has units of force. Thus, the product of Q x A represents the maximum drag force a body could experience due to pressure. In reality, no real body experiences the maximum dynamic pressure over its entire face; the pressure always falls off with distance away from the stagnation point (as discussed above). Therefore, the total drag force experienced by a real body due to pressure will be less than this maximum. The real pressure force will be some fraction of the maximum possible and this fraction is customarily given the name drag coefficient (C_d). The total pressure drag force exerted on a body of cross-section area (A), by air flowing at velocity (v), is:

$$F = Q \times A \times C_d \tag{4}$$

where we recall from (Eq.1) that $Q = \frac{1}{2}\ p\ v^2$. In this equation for aerodynamic drag, all of the information about the shape of the object and the shape of air flow around it is contained in the drag coefficient.

As we have seen however in the discussion above, there are many qualitatively different flow regimes that correspond for example to fully laminar, circulating, turbulent, transonic, supersonic, and hypersonic flow. What we would expect (and what is seen experimentally) is that there is a different drag coefficient for the spherical shape that corresponds to each of these different flow regimes. In most cases, the drag coefficient varies between about 0.1 and a 0.6.

For very low Reynolds Numbers the drag coefficient will actually be greater than one because under these conditions very little of the drag is due to pressure and almost all of it is due to viscosity.

What has not been discussed yet is the idea that in order to create lift on a body it is necessary that there be an imbalance of pressure between the top of the body and the bottom. This can most easily be seen when the body is an airfoil section. On an airfoil the distance an air molecule must follow when flowing over the top of the airfoil is significantly greater than the distance a molecule would flow when passing under the bottom of the airfoil. This difference in path length results in a difference in density and therefore a difference in pressure, between the top and bottom. On a sphere, all paths around the surface are great circles and they are the same length. Therefore, the airflow cannot generate a difference in pressure between one side and another. This leads to the simple realization that a purely translating sphere cannot generate lift.

Nevertheless, it is possible for a sphere to generate lift if the sphere is rotating. As discussed above, viscosity is a force that tends to make air molecules stick to the surface they are in contact with. If a sphere is rotating at the same time it is translating the viscous forces will tend to move air from one side of the sphere to the other thereby creating a density and pressure differential. Any effect that results in creating a pressure differential from one side of the sphere to the other and is perpendicular to the flight velocity will create lift. This effect is referred to as the Magnus effect and is employed whenever topspin or backspin is placed on a tennis ball, for example. Although this effect is real, it is usually small and a relatively inefficient means of generating lift on a sphere.

Summary and Conclusion

Any solid body with an impermeable skin will deflect air flow around itself when that body is in motion relative to the air. A full understanding of aerodynamics attempts to explain both the flow patterns of the air molecules around the body and the resulting system of forces imposed on the body. For analytical purposes the sphere is practically the simplest possible geometric shape to consider because it always presents the same geometry to a uniform airflow. Even so, the details of airflow around a sphere can exhibit high levels of complexity; the flow patterns change with differences in Reynolds Number, and Mach Number. This leads to more than a half-dozen distinct flow regimes characterizing airflow around a sphere. Fortunately, the net effect of airflow around a non-rotating sphere is to simply produce aerodynamic drag force acting in the same direction as the free stream velocity. The relative magnitude of the drag force changes quantitatively in the different flow regimes and is characterized by a non-dimensional parameter known as the drag coefficient. Due to its high level of geometric symmetry, a translating sphere is unable to produce aerodynamic lift. A rotating sphere, however, is capable of producing low levels of lift inefficiently through a viscous interaction with air molecules known as the Magnus effect.