2.5

Geometric Relations of a Sphere

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It is possible to define a sphere in several different ways. The simplest yet perhaps least useful is what Figure 1 shows.[1] Indeed, it requires no additional words until one becomes interested in two related subjects: first, its "other side" that cannot be seen, and second, a more precise description suitable for mathematical and geometric studies.

What About the Third Dimension?

What would one find if we could view this white object from the side or its opposite end? Perhaps it is a giant ice-cream cone viewed here from the top or perhaps a cylinder or other shape. No single, two-dimensional photograph can answer such three-dimensional questions unless a mirror is used within the photograph to simultaneously show another side of the object by reflection.

Figure 1. An Inflated Weather Balloon at Launch

[1] Photo courtesy of Scientificsales Inc., <www.scientificsales.com>

Seldom are there any useful stereoscopic "depth" cues related to the three-dimensional form of most UAP seen at large distances. This is because the separation distance of the eyes within the skull is very small relative to the distance to the UAP. Thus, the poorly discriminated third dimension of the object is usually inferred on the basis of shading of the ambient illumination upon its visible surface or its familiarity.

More Precise Characterizations of a Sphere

A sphere may be defined as:

> *a solid bounded by a spherical surface* (Anon., Pg. 1550, 1958).

However the object in Figure 1 is not a solid in the normal sense of the word. It is a very thin membrane of latex rubber weighing perhaps 500 grams and inflated by a gas to a diameter of about ten feet. So Figure 1 is at best a misleading representation of the above definition. Nevertheless, it does convey a general (and popularized) idea of what a sphere is.

When the physical nature of the sphere (solid or not) is not of concern then a sphere can be defined as:

> a three-dimensional object or form whose surface is defined by a constant radius[2] R from its center.[3]

Letting D = the diameter[4] of the sphere, S = the surface area, $\pi = 3.14159$, and V = the volume of the sphere; Then:

$$D = 2R \qquad (1)$$

$$S = 4 \pi R^2 = \pi D^2 = 12.57 R^2 \qquad (2)$$

$$V = 4/3 \pi R^3 = 1/6 \pi D^3 = 4.189 R^3 \qquad (3)$$

Finally, if we disregard the idea that the sphere is an "object" we find a sphere to be:

> the set of all points in three dimensional Euclidian space that are located at the same distance R from its center. (Mathworld, 2009)

We shall not be concerned here with the distinctions that geometers and topologists make of 3-sphere and 2-sphere space nor with equations for a sphere expressed in Cartesian, spherical

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2 A radius is a straight line with one end at the center of the sphere and the other at the surface. Because R is constant the sphere can be considered a rigid form and not changeable in size.

3 These relations were derived in about 225 BC by Archimedes.

4 A diameter is a straight line twice as long as the radius; it passes through the sphere's center and each end touches the surface at points called antipodes.

coordinates or parametrically.[5] The various geometric characteristics of a sphere have been quantified extensively (Beyer, 1981) and form the basis for spherical trigonometry that undergirds geodesy, terrestrial navigation, extraterrestrial navigation, and other such topics.

Nevertheless, as we will explore in more detail later in this report, a sphere is the only three-dimensional form that remains invariant when pitched, yawed, or rolled about its center. Put another way and speaking more practically, every other rigid object or form will appear different when pitched, yawed, and/or rolled while the eye point is fixed. Whether this difference actually can be discriminated visually or photographically will depend on the sensitivity of the sensing system. Now we will progress to several subjects that are related to spheres.

Sphere and a Plane. Two interactions are possible. Any plane that passes through a sphere is defined by a circle. The largest circle occurs when the plane is coincident with the sphere's diameter. When the plane passes through the center of the sphere the circle is maximum in area and is called a great circle. Earth's equator is a great circle. It can also be shown both graphically and mathematically that two great circles will meet each other in precisely two antipodal points. On the other hand if the plane is tangent to (just touching) the sphere then a point is defined at their intersection.

When it is recognized that the shortest distance between any two points on the surface of a sphere is along a great circle joining the two points the importance of great circles becomes more apparent. Great circles become analogous to the straight lines drawn on a flat (plane) surface to define a polygon and they very often define the flight path of airplanes flying great distances, e.g., across the oceans because great circle routes tend to save on fuel.

The Lune. Considering a flat plane the first and indeed simplest polygon is a *triangle* (a figure made up of three straight lines that intersect one another). There is no such thing as a two-sided polygon. However two "straight" lines on the surface of a sphere can form a "*lune*." Figure 2 illustrates this interesting geometric shape. It is defined as the enclosed area on the surface of a sphere when any two great circles meet in two antipodal spots. Considering the Earth as a sphere, the north and south poles are vertices or antipodal points.

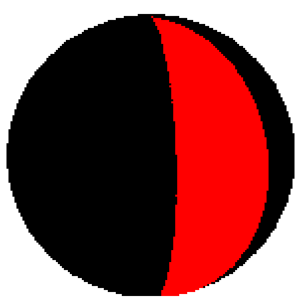

Figure 2. The red area is a lune

[5] See http://mathworld/Wolfram.com (2009) and www.uwgb.edu/dutchs/mathalgo/CIRCSPH.HTM for a fuller discussion of these subjects and mathematical formulae. Wolfram mathworld (op cit) discourages the use of the term “sphere” to refer to the interior of a sphere but prefers the term “ball.” This is because, he maintains, “sphere” (should) refer only to the surface because the “usual sphere is a two-dimensional surface.

When balloons were first manufactured (also as were parachutes) out of fabric they were constructed of many gores or lunes attached together along their edges. Today most balloons are made of highly elastic latex rubber in many flat sheets that are bonded together.

We shall make an arbitrary distinction in the present collection of papers as to whether or not the reported object is a sphere. That is, we must agree to accept the witness's description that he saw a sphere even though it might have been an oblate spheroid or even an oval with a 99:100 orthogonal axis ratio (or less). When a flight crew sees a generally spherically shaped object while traveling at high speed and has only seconds to look at it, judge its distance, trajectory, and potential for collision (or other threat to their safety) it is a mute point whether it is a sphere. For practical purposes we shall adopt a more general definition of the term "sphere" here.

References

Beyer, W.H., CRC Standard Mathematical Tables. CRC Press, Inc., Boca Raton, FL, Pp. 128, 259, 262, 355, 1981.

http://mathworld.wolfram.com/Sphere.html, 2009.